Bandpass Filter With Adjustable Q Has Constant Maximum Gain

Herminio Martínez, Joan Domingo, Juan Gámiz, and Antoni Grau

herminio.martinez@upc.es joan.domingo@upc.es

juan.gamiz@upc.es antoni.grau@upc.es

EUETIB and Technical University of Catalonia (UPC), Barcelona, Spain

Some applications, such as audio equalizers, require bandpass filters with a constant maximum gain (at the center frequency, ω_o), independent of the selected quality factor Q. But in a lot of well-known filter structures—such as Sallen-Key, MFB, state variable, and Tow-Thomas—when you adjust the quality factor of the second-order cells, the maximum gain changes.

This is evident from the normalized second-order transfer function of a bandpass filter:

$$H_{BP}(s) = K\frac{\left(\frac{s}{\omega_o}\right)}{\left(\frac{s}{\omega_o}\right)^2 + \frac{1}{Q}\left(\frac{s}{\omega_o}\right) + 1} \qquad (1)$$

where K is the gain constant of the bandpass filter and $s = \sigma + j\omega$ is the Laplacian operator. When the frequency of the input signal is ω_o, the maximum gain of the filter $|H_{BP}(\omega)|_{max}$ equals the product KQ. Therefore, if the quality factor changes, so does the maximum gain.

Figure 1 shows a filter structure that creates a gain constant, K, that's inversely proportional to the chosen quality factor. As a result, when Q is modified, K is also modified so that the product KQ remains constant, as does the maximum gain at ω_o.

The filter consists of a twin-T cell, where quality factor can be adjusted, plus a differential stage (op-amp OA_3 and the four resistors labeled R5). The output of this stage, $V_{OUT}(t)$, is the difference between the input signal to the filter and the output $V_{BR}(t)$ of the twin-T network.

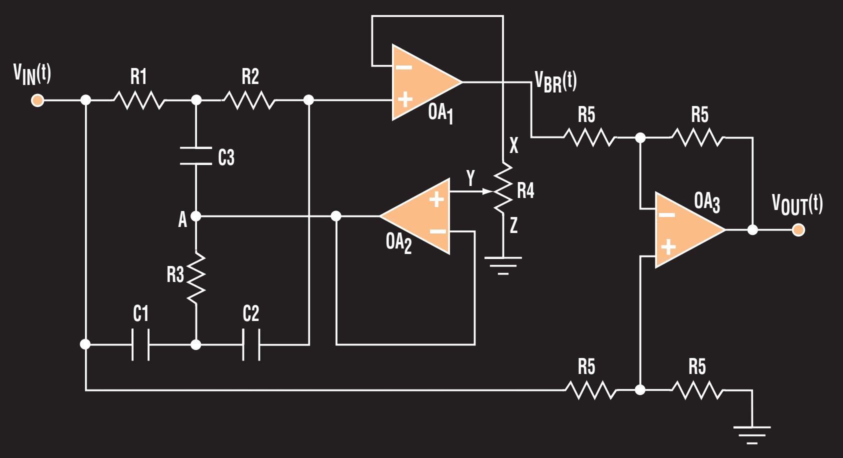
If capacitors C1 and C2 are equal, with a value C, capacitor C3 will equal 2C. Resistors R1 and R2 are equal, with a value of R, and R3 will equal R/2. In this condition, the twin-T circuit behaves as a notch filter, which has a transfer function of:

$$H_{BR}(s) = \frac{V_{BR}(s)}{V_{IN}(s)} = \frac{(RCs)^2 + 1}{(RCs)^2 + 4RC(1-m)s + 1} \qquad (2)$$

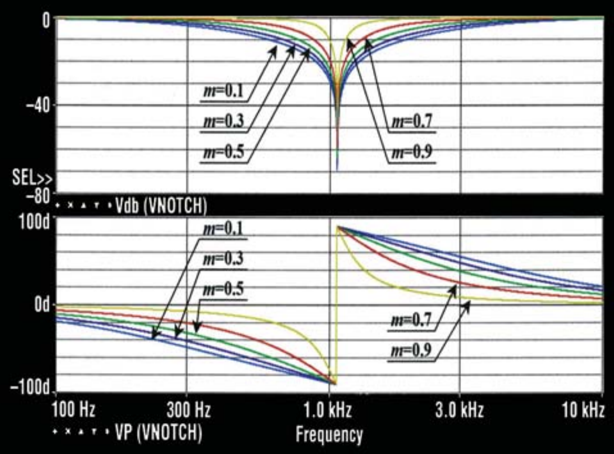
And, the global circuit (output $V_{OUT}(t)$) is a bandpass filter with a transfer function of:

$$H_{BP}(s) = \frac{V_{OUT}(s)}{V_{IN}(s)} = \frac{4RC(1-m)s}{(RCs)^2 + 4RC(1-m)s + 1} \qquad (3)$$

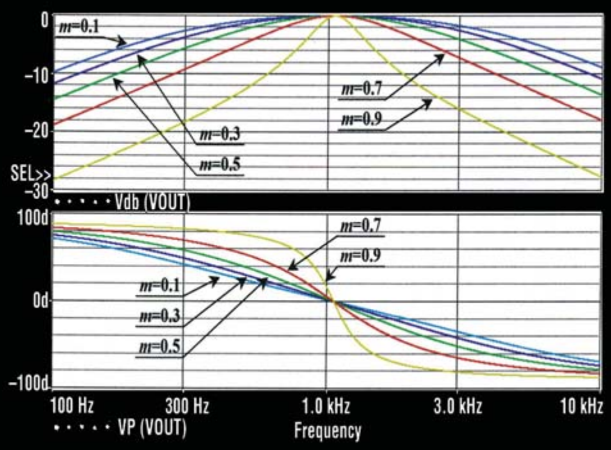
where m is the feedback factor in the



1. This bandpass filter has adjustable Q and constant maximum gain. It's composed of a twin-T cell whose quality factor is adjustable, plus a differential stage (op-amp OA_3 and the resistors designated R5).



2. Here's a set of magnitude and phase Bode plots of the filter for the notch output $V_{BR}(t)$, using values of m between 0.1 and 0.9.



3. This set of magnitude and phase Bode plots of the filter is for the bandpass output $V_{OUT}(t)$, using values for m between 0.1 and 0.9.

twin-T cell. If R_{XY} is the portion of the resistance that exists between the upper terminal (point X) and the cursor (point Y) of potentiometer R4, and R_{YZ} is the portion of resistance that exists between the cursor and the bottom terminal (point Z), the m is:

$$m = \frac{R_{YZ}}{R_{XY} + R_{YZ}} = \frac{R_{YZ}}{R4} \qquad (4)$$

Comparing Equation 3 with the respective normalized transfer function of a bandpass filter *(Equation 1)*, the central frequency of the filter ω_o (coincident with the transmission zero of the twin-T network) is:

$$\omega_o = \frac{1}{RC} \qquad (5)$$

and the quality factor and gain constant are:

$$Q = \frac{1}{4(1-m)} \qquad (6)$$

$$K = \frac{1}{Q} = 4(1-m) \qquad (7)$$

Then, the maximum gain $|H_{BP}(\omega)|_{max}$ at $\omega = \omega_o$ always remains constant, and equal to 1 (0 dB), independently of the quality factor Q. The minimum quality factor is 1/4 (for m = 0, corresponding to the cursor of the potentiometer connected to ground), and the maximum is (theoretically) infinite. In practice, it's convenient not to have quality factors beyond 50. In most applications, typical quality factors range from 1 to 10. To fix ω_o, we can determine the value of R and C thanks to Equation 5. The value of R5 in the differential amplifier isn't critical for the center frequency and the Q factor. The typical value for these resistors is in the medium-kΩ range.

As a particular example, Figure 2 shows a set of magnitude and phase Bode plots of the filter for the notch output, $V_{BR}(t)$, for values of m between 0.1 and 0.9. Figure 3 shows the Bode curves of the bandpass output, $V_{OUT}(t)$, for the same values of m. In both figures, the frequency f_O is 1061 Hz.

To minimize the frequency shift where the transmission zero exists, and improve the accuracy of the circuit, use precision resistors. These include metallic film resistors with tolerances of 1% or better. The capacitors could be mica, polycarbonate, polyester, polystyrene, polypropylene, or Teflon devices. In any case, avoid the use of carbon resistors and electrolytic, tantalum, and even ceramic capacitors. **ED Online 10684**

HERMINIO MARTÍNEZ, assistant professor with the Department of Electrical Engineering at UPC, received a BSc in electrical engineering, an MSc in electronics engineering, and a PhD in electronics from UPC.

JOAN DOMINGO is an associate professor with UPC's Systems Engineering, Automation and Industrial Informatics (ESAII) Department. He received a BSc in electrical engineering and an MSc in electronics engineering from UPC and a PhD in electronics from Barcelona University.

JUAN GÁMIZ is a professor in UPC's ESAII Department. He earned a BSc in electrical engineering and an MSc in electronics engineering from UPC and a PhD in electronics from Barcelona University.

ANTONI GRAU is a professor at the School of Informatics of Barcelona. He received an MS and a PhD in computer science from UPC.